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Discussion Paper No. 25
October 2004

ISSN 1795-0562

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# Aging in a Currency Union with Endogenous Growth

## Abstract

This paper considers the effects of aging in a currency union with endogenous growth. It is shown that mortality, labour supply and the increase in private finance in medical care have no effect on inflation and therefore pose no problems for monetary integration. On the other hand, the demand for medical care speeds up inflation. Hence, if a currency union accepts a new member with a relatively high demand for medical care, its inflation rate increases, but the growth rate and welfare decrease everywhere in the enlarged union. This tendency cannot be indefinitely outweighed by privatizations, for there exists a growth-maximizing proportion of private finance in medical care.

**JEL Classification**: J50, O40

**Keywords**: growth, inflation, congestion, aging.

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# 1 Introduction

Since 2002, over 300 million Europeans in 12 countries have used Euro as a currency of exchange. The demographic variation among these countries is however large. Total Fertility Rate (TFR) is relatively high for some members of the EMU (e.g. 1.9 for Ireland, 1.7 for France and Finland) and relatively low for others (e.g. 1.3 for Germany and Greece and 1.2 for Italy and Spain).[1] Does this prose a problem for the stability of the EMU? For some new members of the EU, TFR is even lower (e.g. 1.2 for Estonia, Slovenia and Czech Republic, 1.1 for Latvia).[2] Should the EMU restrict the membership of these? This paper attempts to answer these questions.

Rising longevity affects national savings and intensifies the competition of resources between consumption, investment and the health needs of the elderly. Recent empirical work indicates that it may even slow down economic growth.[3] Zhang et.al. (2003) explain this through the assumption that aging changes the preferences of the median voter with regard to taxation for public education.[4] In this study, we offer another explanation as follows. Because a currency union compels economies into a sub-optimal tax system by imposing a uniform inflation rate, its establishment may together with the consequences of aging retard growth.

Aging has several and even opposing effects on growth. Lower mortality raises the fraction of population living in retirement substantially. In industrial countries, the proportion of working population falls and is expected to fall even further in the next few decades. The return on individual investment in human capital depends positively on remaining active years. Postponing retirement raises the proportion of working individuals, the returns on human capital and the sustainable growth rate. Because longer life expectancy

[1] United Nations (2000).

[2] United Nations (2000).

[3] Using the cross-section data set in Barro and Wolf (1989), Zhang et.al. (2003) reports that as the initial life expectancy rises from below 60 to 69, the investment ratio and the growth rate increase substantially; but when the initial life expectancy rises to 70 and over, the investment ratio and the growth rate drop, although their levels are still higher than in countries with low life expectancy.

[4] The median voter is willing to increase the tax rate for public education at low longevity, but beyond some level of longevity, further declines in mortality may lead the median voter to lower the tax rate. Hence, the human capital accumulation may rise initially but may eventually fall.

increases the proportion of retirees, it speeds up growth only if the working period is simultaneously expanded.[5] In developed countries, old-age social security programs are under increasing pressure from mortality decline and many recent proposals call for reducing public funding for social security.

In this study, we examine endogenous growth in a monetary economy by a model of a dynastic family which accumulates (human) capital through saving and holds money for transactions purposes. Each family contains both young (working) and elderly (retired) people. Given the discussion above, we characterize aging by the following parameters:

(*i*) Mortality declines. This is equivalent to the decrease in the rate of time preference [Cf. Blanchard and Fischer (1989), pp. 115-143].

(*ii*) The proportion of working population declines. This is equivalent to the decrease of the productivity of (human) capital.

(*iii*) The demand for medical care increases. This can be financed privately or from the government budget.

In this study, a key feature of the analysis is the central role assigned to the congestion of medical services. Following Palokangas (2003), we show that a specific form of congestion is necessary for persistent growth. Inflation has then two effects on the growth rate. First, its increase provides the government with more seigniorage and thereby helps to supply more medical care. This promotes private output, private saving, capital accumulation and economic growth. On the other hand, a higher inflation rate leads to higher transaction costs in the private sector, lower income, lower capital accumulation and slower growth. Where these two opposed effects exactly match, the inflation rate is optimal and the growth rate maximal. A number of endogenously-growing economies produce the same composite good. A currency union compels the same inflation rate for all of its members.

The remainder of this paper is organized as follows. Section 2 introduces the institutional specifications on which the study is based. Section 3 presents the optimal behaviour of a private family, on which in section 4 the equilibrium of the private sector is based. Section 5 characterizes the

[5] Cf. Echevarria (2003).

structure of the public sector and section 6 constructs optimal policy rules for a rational government of a single economy. Finally, section 7 examines the effect of currency unions in this environment.

# 2 The setting

*(i) Production and consumption.* We aggregate all goods in the economy into a single good, the price of which is $p$. There are two assets, money and capital. There is a fixed number $J$ of similar private families who save, invest in capital, hold money for transaction purposes and produce goods from capital. All private families benefit from medical care $G$. One unit of medical care is produced from one unit of the good. A fixed share $\beta \in [0, 1]$ of medical care $G$ is financed by service payments from customers and the rest through the government's budget.

Output $Y$ is produced from capital stock $K$ according to

$$Y = \xi K, \tag{1}$$

where $\xi$ is a parameter. The older people, the less of these takes part in production and the lower $\xi$. The intertemporal utility function is given by

$$\int_0^\infty U(C, G)e^{-\rho t}dt = \int_0^\infty U(c, g)K^{1-\sigma}e^{-\rho t}dt \text{ with}$$
$$U(c, g) = (c + \alpha g)^{1-\sigma}/(1 - \sigma), \;\; g \doteq G/K \text{ and } c \doteq C/K, \tag{2}$$

where $t$ is time, $C$ consumption, $G$ medical care, $\rho > 0$ the constant rate of time preference, $\sigma \in (0, 1) \cup (1, \infty)$ the inverse of the constant intertemporal elasticity of substitution, and $\alpha$ the constant marginal rate of substitution between consumption and medical care when the level of instantaneous utility $U$ is kept constant. The higher mortality, the higher $\rho$. The older people, the more they need medical care and the higher subjective price $\alpha$ they are ready to pay for this in terms of consumption.

*(ii) Congestion.* Congestion results from the existence of many families. As distinct from aggregate output $Y$ and aggregate capital $K$, we denote a single family's output and capital by $y$ and $k$. With congestion, a single family assumes it will get the more services $\widetilde{G}$ from medical care $G$ the larger its

share $k/K$ of aggregate capital stock $K$. In line with Fischer and Turnovsky (1998), we specify this as follows:

$$\widetilde{G} \doteq (k/K)^{\delta} G = (k/K)^{\delta} gK = (k/K)^{\delta-1} gk \text{ with } 0 < \delta \leq 1, \tag{3}$$

where $\delta$ is a parameter. When congestion is *proportional*, $\delta = 1$, the family assumes that it receives medical services $\widetilde{G}$ in direct proportion to its capital stock $k$. When congestion is *partial*, $0 < \delta < 1$, the family assumes that it receives less $\widetilde{G}$ than in proportion to $k$. We ignore the case of no congestion, $\delta = 0$, where medical care $G$ is a non-rival and non-excludable public good available equally to each family independent of the size of the economy.

The family takes medical care $G$, aggregate capital stock $K$ and $g \doteq G/K$ as given. It perceives the true production function (1) and the true utility function (2), but so that macroeconomic variables $G$, $K$ and $Y$ are replaced by microeconomic variables $\widetilde{G}$, $k$ and $y = \xi k$. Because the families are similar, the consumption-capital ratio $c$ must be the same for the whole economy and a single family. Given (3), we then obtain a single family's perceived utility function as follows:

$$\int_0^{\infty} U(ck, \widetilde{G}) e^{-\rho t} dt = \int_0^{\infty} \left[ c + \alpha \Big( \frac{k}{K} \Big)^{\delta-1} g \right]^{1-\sigma} k^{1-\sigma} e^{-\rho t} dt. \tag{4}$$

*(iii) Tax evasion.* A family is able to hide income at some cost.[6] Let $py$ is the family's total income, $qpy$ hidden income and $(1-q)py$ observed income, where $0 \leq q \leq 1$. We assume that the level of income does not affect the family's ability to conceal income, but that such activity is subject to increasing costs. The real administrative cost of hiding income, $Z$, is then linear homogeneous with respect to total real income $y$ but increasing and strictly convex with respect to the ratio $q$ of hidden to total income. With all profits revealed, $q = 1$, there is no administrative cost $Z = 0$. Given these assumptions, the following (real) cost function can be established:

$$Z = z(q)y, \quad z' > 0, \quad z'' > 0, \quad z(0) = 0, \quad z \doteq Z/(py), \tag{5}$$

where $z$ is the ratio of administrative cost to total income.

[6] This assumption is needed to produce a distortion in the public sector, which gives the government the incentive to use seigniorage.

*(iv) Transaction technology.* We introduce money as an intermediary good which reduces transaction costs.[7] One unit of transaction services is produced from one unit of the composite good. The requirement for real transaction services, $V$, is an increasing function of real expenditure

$$E \doteq ck + \dot{k} + \beta\widetilde{G}, \tag{6}$$

which consists of consumption $ck$, investment $\dot{k} = dk/dt$ and the private cost for medical services $\widetilde{G}$, $\beta\widetilde{G}$, and a non-increasing function of the real level of money balances, $M/p$, which is the ratio of the money supply $M$ and the price level $p$, $V = \mathcal{V}(E, M/p)$. This function is also linearly homogeneous, i.e. a proportional increase in both real expenditure $E$ and real money stock $M/p$ increases $V$ by the same proportion.

To obtain a stable demand function for money, we assume furthermore that $\mathcal{V}$ is strictly concave and thrice differentiable.[8] The function $\mathcal{V}$ can then be transformed into the form $V = v(m)E$, where $m \doteq M/E$ is the money-expenditure ratio and $v(m) \doteq \mathcal{V}(1, m)$ is a thrice differentiable function with $v'' > 0$. Finally, we assume that there is a bliss point $\bar{m}$ for the money-expenditure ratio with $m \leq \bar{m}$ and $v(\bar{m}) = v'(\bar{m}) = 0$. Defining the rate of investment $\phi \doteq \dot{k}/k \geq 0$, we can summarize the transaction technology as:

$$\begin{aligned} &0 \leq m \doteq M/E \leq \bar{m}, \quad 0 \leq v(m) = V/E < 1, \\ &v' \leq 0, \quad v'' > 0, \quad v(\bar{m}) = v'(\bar{m}) = 0. \end{aligned} \tag{7}$$

# 3 The families

In the family's steady state, the ratios $c = C/K$, $g = G/K$ as well as the ratio of medical services (3) to private consumption $C = ck$, i.e. $(k/K)^{\delta-1}g/c$, must be constant. This means that the term $k^{\delta-1}$ must be constant as well. Capital stock $k$ and output $y$ are then constant for $0 < \delta < 1$. If $\delta = 1$, $k$ is undetermined, the rate of investment $\phi = \dot{k}/k$ can be positive and output can grow at a positive rate. We summarize:

[7] This follows Palokangas (2003). Feenstra (1986) shows that under certain conditions, the approach of placing money in the utility function is equivalent to this approach.

[8] Thrice differentiability is needed for the differentiability of the elasticity $\varepsilon$ in (15).

**Proposition 1** *Persistent growth is possible only with proportional congestion $\delta = 1$. With partial congestion, $0 < \delta < 1$, there is no such growth.*

With proportional congestion, the average product of capital, $y/k$, is constant, there is no equilibrium for capital stock and capital grows indefinitely. With partial congestion, the average product of capital is decreasing, there is an equilibrium for capital stock and there is no growth in capital.

An family's budget constraint is given by

$$(1-x)(1-q)y + qy = E + iM/p + V + Z, \tag{8}$$

where $y$ is its output (= total real income), $qy$ its hidden income, $(1-q)y$ its revealed income, $x$ the tax rate, $E$ its real expenditure, $M$ its money supply, $iM/p$ the depreciation in its real balances $M/p$ due to the inflation rate $i$, $V$ its purchase of transaction services and $Z$ its costs in tax evasion. Given (3), (5), (6), (7), $\delta = 1$, $y = \xi k$ and $\phi \doteq \dot{k}/k$, the constraint (8) takes the form

$$\begin{aligned}&[(1-x)(1-q) + q - z(q)]\xi = [(1-x)(1-q) + q - z(q)]y/k \\ &= (1 + im + v)E/k = [1 + im + v(m)](c + \phi + \beta g).\end{aligned} \tag{9}$$

An family maximizes its utility (4) subject to its budget constraint (9) and its capital accumulation $\phi \doteq \dot{k}/k$ by the consumption-capital ratio $c$, the rate of investment, $\phi$, the ratio of hidden to total income, $q$, and the money-expenditure ratio $m$, given the inflation rate $i$, the tax rate $x$, medical care $G$, the aggregate capital stock $K$ and $g = G/K$.

Since our purpose is to examine public policy in an economy with persistent growth, then, given proposition 1, we can focus wholly on the case $\delta = 1$. Given $\delta = 1$, $\widetilde{G} = G$ (4) and (9), the family maximizes its utility

$$\int_0^\infty U(c,g)k^{1-\sigma}e^{-\rho t}dt = \int_0^\infty \frac{e^{-\rho t}}{1-\sigma}(c + \alpha g)^{1-\sigma}k^{1-\sigma}dt$$

subject to the budget constraint (9) and capital accumulation $\phi \doteq \dot{k}/k$ by variables $c$, $\phi$, $z$, $q$ and $m$, given $i$, $x$, $G$, $K$ and $g \doteq G/K$. The Lagrangean of this maximization is given by

$$\begin{aligned}\Psi &= (c + \alpha g)^{1-\sigma}k^{1-\sigma}/(1-\sigma) + \mu\phi k \\ &\quad + \omega\big\{[(1-x)(1-q) + q - z(q)]\xi - [1 + im + v(m)](c + \phi + \beta g)\big\},\end{aligned} \tag{10}$$

where $\omega$ is a Lagrangean multiplier and variable $\mu$ evolves such that

$$\dot{\mu} = \rho\mu - \partial\Psi/\partial k = (\rho - \phi)\mu - (c + \alpha g)^{1-\sigma}k^{-\sigma}, \quad \lim_{t\to\infty} \mu k e^{-\rho t} = 0. \tag{11}$$

# 4 The equilibrium of the private sector

The maximization of the Langrangean (10) by $q$ leads, by duality and by the properties of the function (5), to the definition

$$\pi(x) \doteq \max_q [(1 - x)(1 - q) + q - z(q)], \quad \pi' < 0, \quad \pi'' > 0. \tag{12}$$

Given this definition, we obtain the tax base $T$ and the elasticity of the tax base with respect to the tax rate $x$, when capital $k$ and medical care intensity $g$ are kept constant, as follows:

$$T = (1 - q)y = -\pi'(x)\xi k, \quad \eta(x) \doteq -\frac{x}{T}\frac{\partial T}{\partial x} = -x\frac{\pi''(x)}{\pi'(x)} > 0. \tag{13}$$

Given (4), the maximization of the Lagrangean (10) by $m$ yields

$$v'(m) = -i. \tag{14}$$

From this and (7) it follows that the money-expenditure ratio, $m$, is a function of the inflation rate $i$ only:

$$m(i), \;\; m' \doteq \frac{dm}{di} = -\frac{1}{v''} < 0, \;\; \varepsilon(i) \doteq -\frac{im'}{m} = \begin{cases} > 0 & \text{for } m < \bar{m}, \\ = 0 & \text{for } m = \bar{m}, \end{cases} \tag{15}$$

where $\varepsilon$ is the elasticity of the demand for money with respect to the inflation rate $i$, when real expenditure $E$ is kept constant.

The maximization of the Lagrangean (10) by $c$ and $\phi$ yields

$$\begin{aligned} \partial\Psi/\partial c &= (c + \alpha g)^{-\sigma}k^{1-\sigma} - [1 + im + v(m)]\omega = 0, \\ \partial\Psi/\partial\phi &= \mu k - [1 + im + v(m)]\omega = 0. \end{aligned} \tag{16}$$

Output $y = \xi k$, consumption $ck$ and medical services $\tilde{G} = gk$ are now in fixed proportion to capital stock $k$, which is the family's only state variable. Consequently, the system jumps immediately to the steady state and there are no transitional dynamics. Given the first-order conditions (16), we obtain

$$\mu = (1 + im + v)\omega/k = (c + \alpha g)^{-\sigma}k^{-\sigma}.$$

This implies that terms $k^{-\sigma}$ and $\mu$ grow at the same rate, $\dot{\mu}/\mu = -\sigma\,\dot{k}/k = -\sigma\phi$. This, (11) and (14) produce

$$\rho + (\sigma - 1)\phi = (c + \alpha g)^{1-\sigma} k^{-\sigma}/\mu = c + \alpha g. \tag{17}$$

Inserting $\delta = 1$, (12) into (9) and solving for $\phi$, we obtain

$$\dot{k}/k = \phi = \xi\pi(x)/[1 + im + v(m)] - c - \beta g. \tag{18}$$

A balanced-growth equilibrium exists in the model, because the model is proportional to the state variable $k$. Inserting (17) into (18) yields:

$$\phi = \Phi(x, i, g, \rho, \xi, \alpha, \beta) \doteq \frac{1}{\sigma}\Big[\frac{\xi\pi(x)}{1 + im + v(m)} + (\alpha - \beta)g - \rho\Big]. \tag{19}$$

Because in equilibrium the consumption-capital ratio $c$, the rate of investment $\phi$ and the money-expenditure ratio $m$ are kept constant, given (7), money supply $M$ and the price level $p$ grow at the same rate, $\dot{p}/p = \dot{M}/M$. Hence, by increasing the quantity of money per family, $M$, at a fixed rate, the government can control the inflation rate $i \doteq \dot{p}/p = \dot{M}/M$.

# 5 Governments

Because the families are similar, aggregate capital is given by $K = Jk$, where $J$ is the number of families. Since the public expenditures on medical care, $(1-\beta)G$, are financed by taxes $xT$ and seigniorage $iM/p$ from all $J$ families, the government's budget constraint is $(iM/p + xT)J = (1-\beta)G$. Given (7), (12), (13), (15), (19), $g = G/K$ and $K = Jk$, this constraint also reads as:

$$\begin{aligned}\Upsilon(i, g, x, \xi, \beta) &\doteq \big[(iM/p + xT)J - (1-\beta)G\big]/(\xi K) \\ &= (c + \phi)im/\xi - x\pi' - (1-\beta)g/\xi \\ &= \frac{\pi(x)im(i)}{1 + im + v(m)} - x\pi'(x) - (1-\beta)\frac{g}{\xi} = 0. \end{aligned} \tag{20}$$

We assume that the economy is on the increasing part of the Laffer curve. This means that if the government budget is initially balanced, $\Upsilon = 0$, and the inflation rate $i$ is kept constant, then an increase in the tax rate $x$ produces a budget surplus $\Upsilon > 0$. Given (12), (13) and (20), this implies

$$\frac{\partial\Upsilon}{\partial x} = \pi'\Big[\frac{im}{1 + im + v} - 1 + \eta\Big] > 0, \quad 1 > \eta + \frac{im}{1 + im + v}. \tag{21}$$

The government maximizes welfare by the tax rate $x$, the inflation rate $i$ and medical care intensity $g \doteq G/K$, given the budget constraint (20). We can equivalently express the budget constraint (20) in terms of the tax rate and assume that the government maximizes welfare by $i$ and $g$, given this tax function. Differentiating (20) totally, and noting (13), (15) and (21), we obtain the tax rate as the following function of the other policy variables $(i, g)$ and the parameters $\xi$ and $\beta$:

$$
\begin{aligned}
&x(i,g,\xi,\beta), \quad \frac{\partial x}{\partial \xi} = (\beta-1)\frac{g}{\xi^2}\Big/\frac{\partial \Upsilon}{\partial x} < 0, \quad \frac{\partial x}{\partial \beta} = -\frac{g}{\xi}\Big/\frac{\partial \Upsilon}{\partial x} < 0,\\
&\frac{\partial x}{\partial g} = -\frac{\partial \Upsilon}{\partial g}\Big/\frac{\partial \Upsilon}{\partial x} = \frac{1-\beta}{\xi\pi'}\Big[\frac{im}{1+im+v} - 1 + \eta\Big]^{-1} > 0,\\
&\frac{\partial x}{\partial i} = -\frac{\partial \Upsilon}{\partial i}\Big/\frac{\partial \Upsilon}{\partial x} = \left[\frac{m}{(1+im+v)^2} - \frac{m+im'}{1+im+v}\right]\pi\Big/\frac{\partial \Upsilon}{\partial x}\\
&\quad = \frac{m(i)\pi(x)}{[1+im+v(m)]\pi'(x)}\,\frac{\varepsilon(i) + im(i)/[1+im+v(m)] - 1}{\eta(x) + im(i)/[1+im+v(m)] - 1}.
\end{aligned} \tag{22}
$$

Given (13), (18), (19) and (22), the growth rate can then be specified as a function of the policy variables $(i, g)$ and the parameters $(\rho, \xi, \alpha)$ (Appendix):

$$
\begin{aligned}
&\dot{k}/k = \phi(i,g,\rho,\xi,\alpha,\beta) \doteq \Phi(x(i,g,\xi,\beta), i, g, \rho, \xi, \alpha, \beta),\\
&\frac{\partial \phi}{\partial \beta} > 0 \;\Leftrightarrow\; \eta(x) > \frac{v(m(i))}{1+im+v(m)},\\
&\frac{\partial \phi}{\partial g} = \frac{(1-\beta)/\sigma}{im + [\eta(x)-1][1+im+v(m)]} + \frac{\alpha-\beta}{\sigma},\\
&\frac{\partial \phi}{\partial i} = \frac{\partial \Phi}{\partial x}\frac{\partial x}{\partial i} + \frac{\partial \Phi}{\partial i} = \frac{\pi(x)m(i)\xi/\sigma}{[1+im+v(m)]^2}\,\frac{\eta(x)-\varepsilon(i)}{1-\eta(x) - im(i)/[1+im+v(m)]},\\
&\frac{\partial^2 \phi}{\partial g^2} < 0 \;\Leftrightarrow\; \frac{\partial^2 \phi}{\partial g \partial i}\bigg|_{\partial\phi/\partial i = 0} > 0, \quad \frac{\partial \phi}{\partial \xi} > 0, \quad \frac{\partial \phi}{\partial \alpha} > 0, \quad \frac{\partial \phi}{\partial \rho} < 0,\\
&\frac{\partial^2 \phi}{\partial g \partial \rho} \equiv 0, \quad \frac{\partial^2 \phi}{\partial i \partial \rho} \equiv 0, \quad \frac{\partial^2 \phi}{\partial i \partial \alpha} \equiv 0, \quad \frac{\partial^2 \phi}{\partial g \partial \alpha} = \frac{1}{\sigma} > 0,\\
&\frac{\partial^2 \phi}{\partial g \partial i}\bigg|_{\partial\phi/\partial i = 0}\Big/\frac{\partial^2 \phi}{\partial i \partial \xi}\bigg|_{\partial\phi/\partial i = 0} = \frac{\partial^2 \phi}{\partial g^2}\Big/\frac{\partial^2 \phi}{\partial g \partial \xi},\\
&\frac{\partial^2 \phi}{\partial g \partial i}\bigg|_{\partial\phi/\partial i = 0}\Big/\frac{\partial^2 \phi}{\partial i \partial \beta}\bigg|_{\partial\phi/\partial i = 0} = \frac{\partial^2 \phi}{\partial g^2}\Big/\frac{\partial^2 \phi}{\partial g \partial \beta}.
\end{aligned} \tag{23}
$$

The government maximizes the representative family's utility (4) by $g$ and $i$, given the accumulation of capital (23) and the family's reaction function

(17). It is equivalent to maximizing the Hamiltonian

$$\Lambda = [\rho + (\sigma - 1)\phi(i, g, \rho, \xi)]^{1-\sigma} k^{1-\sigma}/(1-\sigma) + \lambda\phi(i, g, \rho, \xi)k \tag{24}$$

by $g$ and $i$, where variable $\lambda$ evolves according to

$$\dot{\lambda} = \rho\lambda - \partial\Lambda/\partial k = (\rho - \phi)\lambda - [\rho + (\sigma - 1)\phi]^{1-\sigma} k^{-\sigma}, \quad \lim_{t\to\infty} \mu k e^{-\rho t} = 0. \tag{25}$$

# 6 Public policy

Because the model of a family contains only one state variable $k$ and is linearly homogeneous with respect to this, the system jumps immediately to the steady state in which $c$, $\phi$, $g$, $\partial\phi/\partial i$, $\partial\phi/\partial g$ and $\partial c/\partial\phi$ are constants. Given (24), this means that $\lambda$ and $k^{-\sigma}$ must grow at the same rate, $\dot{\lambda}/\lambda = -\sigma\dot{k}/k = -\sigma\phi$. Inserting this into (25) and noting (17), we obtain $\lambda k^{\sigma} = [\rho + (\sigma - 1)\phi]^{-\sigma}$. This and (24) yield

$$\partial\Lambda/\partial\phi = [\rho + (\sigma - 1)\phi]^{-\sigma} k^{1-\sigma}(\sigma - 1) + \lambda k = \sigma\lambda k > 0,$$

which implies

$$\arg\max_{i,g} \Lambda = \arg\max_{i,g} \phi(i, g, \rho, \xi, \alpha, \beta). \tag{26}$$

This result can be rephrased as follows:[9]

**Proposition 2** *A rational government attempts to maximize the growth rate of its economy by its policy variables* $(i, g)$*.*

In an endogenous growth model, the congestion of medical care must be proportional (see proposition 1). This implies that private income is in fixed proportion to capital, and that the government instruments $i$ and $g$ affect welfare only through the growth rate of the economy.

From (23) it follows that

$$(\partial/\partial\rho)\max_{i,g}\phi < 0, \quad (\partial/\partial\xi)\max_{i,g}\phi > 0, \quad (\partial/\partial\alpha)\max_{i,g}\phi > 0,$$
$$(\partial/\partial\beta)\max_{i,g}\phi > 0 \quad \Leftrightarrow \quad \eta > v/(1 + im + v). \tag{27}$$

The first three of these results can be rephrased as:

[9]This proposition is the same as in Palokangas (2003).

**Proposition 3** *A decline in mortality (i.e. a lower $\rho$) and an increase in labour supply (i.e. a bigger $\xi$) promote growth and welfare. Increased demand for medical care (i.e. a bigger $\alpha$) speeds up economic growth.*

A decline in mortality or higher labour supply enable a higher savings rate, which boosts capital accumulation and growth. The higher the relative weight of medical care and lower the relative weight of consumption in a family's preferences, the less incentives the family has to consume and the more to save. A higher savings rate boosts capital accumulation and growth.

The first-order and second-order conditions corresponding to (26) are

$$\frac{\partial \phi}{\partial i}=0,\ \ \frac{\partial \phi}{\partial g}=0,\ \ \frac{\partial^2 \phi}{\partial i^2}<0,\ \ \frac{\partial^2 \phi}{\partial g^2}<0,\ \ \mathcal{J}\doteq\frac{\partial^2 \phi}{\partial i^2}\frac{\partial^2 \phi}{\partial g^2}-\Big(\frac{\partial^2 \phi}{\partial i \partial g}\Big)^2>0. \tag{28}$$

Given this and (23), we reconstruct a result from Palokangas (1997; 2003):

**Proposition 4** ***(Ramsey rule)*** *A rational government chooses he inflation rate $i$ so that the elasticity of money holdings with respect to the inflation rate, $\varepsilon$, is equal to the elasticity of the tax base with respect to the tax rate, $\eta$.*

Since the inflation rate is equivalent to a tax on money, the elasticity of tax revenue with respect to the tax rate, $1-\eta$, must be equal to the elasticity of seigniorage with respect to the inflation rate, $1-\varepsilon$, which yields $\eta=\varepsilon$.

The optimal proportion of private finance in social security is given by $\partial\Lambda/\partial\beta=\partial\phi/\partial\beta=0$. Given (27), this leads to the following result:

**Proposition 5** *To promote growth and welfare, private finance in medical care should be increased (decreased) as long as the elasticity of the tax base, $\eta$, is greater (lower) than the share of transaction cost in total private expenditure, $v/(1+im+v)$.*

In medical care, public finance causes deadweight loss through distorting taxation, but private finance involves transaction costs. There is a trade-off between these two losses.

From (23) and (28) it follows that

$$\frac{\partial^2 \phi}{\partial i \partial g}<0,\quad \frac{\partial^2 \phi}{\partial g \partial \rho}=\frac{\partial^2 \phi}{\partial i \partial \rho}=\frac{\partial^2 \phi}{\partial i \partial \alpha}=0,\quad \frac{\partial^2 \phi}{\partial g \partial \alpha}>0. \tag{29}$$

Differentiating the first-order conditions $\partial\phi/\partial i = 0$ and $\partial\phi/\partial g = 0$ totally and noting (23), (28) and (29), we obtain the partial derivatives

$$\frac{\partial i}{\partial \rho} \equiv 0, \quad \frac{\partial i}{\partial \alpha} = \frac{1}{\mathcal{J}} \frac{\partial^2 \phi}{\partial i \partial g} \frac{\partial^2 \phi}{\partial g \partial \alpha} > 0, \quad \frac{\partial i}{\partial \xi} = \frac{1}{\mathcal{J}} \left[ \frac{\partial^2 \phi}{\partial g \partial i} \frac{\partial^2 \phi}{\partial g \partial \xi} - \frac{\partial^2 \phi}{\partial i \partial \xi} \frac{\partial^2 \phi}{\partial g^2} \right] \equiv 0,$$
$$\frac{\partial i}{\partial \beta} = \frac{1}{\mathcal{J}} \left[ \frac{\partial^2 \phi}{\partial g \partial i} \frac{\partial^2 \phi}{\partial g \partial \beta} - \frac{\partial^2 \phi}{\partial i \partial \beta} \frac{\partial^2 \phi}{\partial g^2} \right] \equiv 0.$$

These results can be rephrased as follows:

**Proposition 6** *Mortality (i.e. $\rho$), labour supply (i.e. $\xi$) and the proportion of private finance in social security (i.e. $\beta$) have no effect on the inflation rate $i$. A higher demand for medical care (i.e. a bigger $\alpha$) speeds up inflation.*

A decline in mortality, higher labour supply or smaller public finance of social security boost saving and economic growth, but do not affect public finance. With a higher demand for medical care, the inflation tax should be raised to collect more seigniorage.

# 7 Currency unions

Finally, we examine the consequences of monetary integration. Proposition 6 has then the following corollary:

**Proposition 7** *Mortality (i.e. $\rho$), labour supply (i.e. $\xi$) and the finance in medical care (i.e. $\beta$) have no effect on monetary integration.*

Let there be two economies, labelled 1 and 2. The economies are similar, except that economy 1 has a larger demand for medical care (i.e. $\alpha_1 > \alpha_2$). Economy 1 has then a lower inflation rate than economy 2,

$$i_1 < i_2. \tag{30}$$

Each economy can exercise fiscal and monetary policy independently of the other, and the growth rates of each may also differ.[10] On the assumption

[10]This property is mainly due to the assumption of a small economy and the exclusion of direct foreign investment from the model, but it helps in analysing monetary integration.

that in both economies fiscal policy is chosen to maximize the welfare of the representative family, we can define the economy-specific growth rates as:

$$\phi^1 \doteq \{\phi|\ \xi = \xi_1,\ \text{fiscal policy optimized in economy 1}\},$$
$$\phi^2 \doteq \{\phi|\ \xi = \xi_2,\ \text{fiscal policy optimized in economy 2}\}.$$

According to proposition 2, the independent monetary policy of economy $j$ should maximize its growth rate $\phi^j$. This yields

$$i_j \doteq \arg\max_i \phi^j \ \text{ for } j = 1, 2. \tag{31}$$

Now assume that the two economies form a currency union so that a common central bank will set a common inflation rate $\iota$ for them. This central bank maximizes a target $\mathcal{W}(\phi^1, \phi^2)$, which is a differentiable and increasing function of the growth rates of economies $\phi^1$ and $\phi^2$. The maximization yields the first-order condition

$$\frac{d\mathcal{W}}{d\iota} = \frac{\partial\mathcal{W}}{\partial\phi^1}\frac{\partial\phi^1}{\partial i}\bigg|_{i=\iota} + \frac{\partial\mathcal{W}}{\partial\phi^2}\frac{\partial\phi^2}{\partial i}\bigg|_{i=\iota} = 0.$$

Given this, the partial derivatives $[\partial\phi^1/\partial i]_{i=\iota}$ and $[\partial\phi^2/\partial i]_{i=\iota}$ must have different signs. In economy $j$ with $[\partial\phi^1/\partial i]_{i=\iota} > 0$, the inflation rate $i$ must be increased from $\iota$ to attain the growth-maximizing level $i_j$ with $[\partial\phi^1/\partial i]_{i=i_j} = 0$, and in economy $j$ with $[\partial\phi^1/\partial i]_{i=\iota} < 0$, $i$ must be decreased from $\iota$ to attain $i_j$. Given (30), this is true only when $i_1 < \iota < i_2$. We have thus obtained our final result:

**Proposition 8** *The establishment of the currency union by two economies will increase (decrease) the inflation rate in the economy with a larger (smaller) demand for medical care, $i_1 < \iota$ ($i_2 > \iota$), and decrease the growth rate and welfare in both economies, $\phi^j\big|_{i=\iota} < \max_i \phi^j = \phi^j\big|_{i=i_j}$ for $j = 1, 2$.*

If institutional differences of potential members are too large, the establishment of a currency union will slow down economic growth and increase the inflation rate in the union.

# 8 Conclusions

This paper examines economies in which the main engine of growth is the existence of a lower limit for the marginal product of (human) capital. Money is introduced as a substitute for transaction services and seigniorage as a substitute for distorting taxation. The governments produce medical care from private sector output and finance this by taxation, seigniorage and direct payments from customers. The establishment of a currency union affects growth and welfare through unifying the inflation rate throughout the member economies. Aging is characterized by three shocks: (*i*) a decline in mortality, (*ii*) a decline in the productivity of capital (through smaller working population) and (*iii*) an increase in the demand for medical care. We are also interested in how the finance of medical care affects the outcome of these shocks. The main findings are the following.

In each member economy, a rational government attempts to maximize the growth rate of its economy by taxation and seigniorage. This can be explained as follows. In an endogenous growth model, the congestion of medical care must be proportional. Private income is then in fixed proportion to capital and the government's policy instruments affect welfare only through the growth rate of the economy. A decline in mortality and an increase in labour supply raise saving and promote growth and welfare. The higher the relative weight of medical care and lower the relative weight of consumption in a family's preferences, the less incentives the family has to consume. This results in a higher savings rate and faster capital accumulation and growth.

To even out the deadweight loss in public finance, a rational government chooses he inflation rate so that the elasticity of money holdings with respect to the inflation rate is equal to the elasticity of the tax base with respect to the tax rate. To promote growth and welfare, private finance in medical care should be increased (decreased) as long as the elasticity of the tax base is greater (lower) than the share of transaction cost in total private expenditure. The publicly financed security system causes deadweight loss through distorting taxation, but the privately financed system involves transaction costs. Hence, the proportion of public finance in medical care should be determined by a trade-off between these two losses.

A decline in mortality, higher labour supply or greater private finance of social security boost saving and economic growth, but do not have any effect on public finance and seigniorage. Hence, mortality and labour supply have no effect on monetary integration. Economies with different rates of mortality and different proportions of working population can have the same inflation rate without any deadweight loss in public finance. With a higher demand for medical care, the inflation tax should be raised to collect more seigniorage. The establishment of the currency union by two economies will then increase (decrease) the inflation rate in economies with a larger (smaller) demand for medical care. This compels sub-optimal taxation and public policy for both economies, which also decreases the growth rate and welfare in both of them. Hence, the need for medical care is the only characteristics of aging which should influence monetary integration. If differences among potential members are too large, the establishment of a currency union will slow down economic growth and increase the inflation rate in the union.

While a great deal of caution should be exercised when a highly stylized growth model is used to draw conclusions about aging, the following judgement nevertheless seems to be justified. A monetary union should not worry about declining labour supply and increasing longevity, which produce only a level effect on income, but on a higher demand for medical care, which boosts inflation and hampers growth. Public finance cuts in medical care is no remedy for this, since it affects the income level but not the growth rate.

# Appendix

From (13), (18), (19) and (22) it follows that

$$
\begin{aligned}
&\dot{k}/k = \phi(i,g,\rho,\xi,\alpha) \doteq \Phi(x(i,g,\xi),i,g,\rho,\xi,\alpha),\\
&\frac{\partial\phi}{\partial g} = \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial g} + \frac{\partial\Phi}{\partial g} = \frac{\xi\pi'/\sigma}{1+im+v}\frac{\partial x}{\partial g} + \frac{\alpha-\beta}{\sigma}\\
&\quad = \frac{(1-\beta)/\sigma}{im+[\eta(x)-1][1+im+v(m)]} + \frac{\alpha-\beta}{\sigma},\\
&\frac{\partial\phi}{\partial i} = \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial i} + \frac{\partial\Phi}{\partial i} = \frac{\xi/\sigma}{1+im+v}\Big[\pi'\frac{\partial x}{\partial i} - \frac{\pi m}{1+im+v}\Big]\\
&\quad = \frac{\pi m\xi/\sigma}{(1+im+v)^2}\left\{\frac{\varepsilon+im/(1+im+v)-1}{\eta+im/(1+im+v)-1} - 1\right\}\\
&\quad = \frac{\pi(x)m(i)\xi/\sigma}{[1+im+v(m)]^2}\,\frac{\eta(x)-\varepsilon(i)}{1-\eta(x)-im(i)/[1+im+v(m)]},
\end{aligned}
$$

$$\frac{\partial\phi}{\partial\xi}=\frac{\partial\Phi}{\partial\xi}+\frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial\xi}>0,\quad \frac{\partial\phi}{\partial\alpha}=\frac{\partial\Phi}{\partial\alpha}=\frac{g}{\sigma}>0,\quad \frac{\partial\phi}{\partial\rho}=-\frac{1}{\sigma}<0,$$

$$\frac{\partial\phi}{\partial\beta}=\frac{\partial\Phi}{\partial\beta}+\frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial\beta}=-\frac{g}{\sigma}-\frac{g/\sigma}{1+im+v}\Big/\Big[\frac{im}{1+im+v}-1+\eta\Big]$$

$$=\frac{g}{\sigma}\frac{\eta-v/(1+im+v)}{1-\eta-im/(1+im+v)}>0\quad\Leftrightarrow\quad\eta>\frac{v}{1+im+v},$$

$$\frac{\partial^2\phi}{\partial g^2}=\frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\frac{\partial x}{\partial g}<0\quad\Leftrightarrow$$

$$\frac{\partial^2\phi}{\partial g\partial\xi}=\frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\frac{\partial x}{\partial\xi}>0\quad\Leftrightarrow$$

$$\frac{\partial^2\phi}{\partial g\partial\beta}=\frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\frac{\partial x}{\partial\beta}>0\quad\Leftrightarrow$$

$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0}=\frac{\pi m\xi/\sigma}{(1+im+v)^2}\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\frac{\partial x}{\partial g}>0\quad\Leftrightarrow$$

$$\left.\frac{\partial^2\phi}{\partial i\partial\xi}\right|_{\partial\phi/\partial i=0}=\frac{\pi m\xi/\sigma}{(1+im+v)^2}\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\frac{\partial x}{\partial\xi}<0\quad\Leftrightarrow$$

$$\left.\frac{\partial^2\phi}{\partial i\partial\beta}\right|_{\partial\phi/\partial i=0}=\frac{\pi m\xi/\sigma}{(1+im+v)^2}\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\frac{\partial x}{\partial\beta}<0\quad\Leftrightarrow\quad\eta'>0,$$

$$\frac{\partial^2\phi}{\partial g\partial\rho}\equiv0,\quad\frac{\partial^2\phi}{\partial i\partial\rho}\equiv0,\quad\frac{\partial^2\phi}{\partial i\partial\alpha}\equiv0,\quad\frac{\partial^2\phi}{\partial g\partial\alpha}=\frac{1}{\sigma}>0,$$

$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0}\Big/\left.\frac{\partial^2\phi}{\partial i\partial\xi}\right|_{\partial\phi/\partial i=0}=\frac{\partial x}{\partial g}\Big/\frac{\partial x}{\partial\xi}=\frac{\partial^2\phi}{\partial g^2}\Big/\frac{\partial^2\phi}{\partial g\partial\xi},$$

$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0}\Big/\left.\frac{\partial^2\phi}{\partial i\partial\beta}\right|_{\partial\phi/\partial i=0}=\frac{\partial x}{\partial g}\Big/\frac{\partial x}{\partial\beta}=\frac{\partial^2\phi}{\partial g^2}\Big/\frac{\partial^2\phi}{\partial g\partial\beta}.$$

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